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Dynamics of sheath evolution in magnetized charge-fluctuating dusty plasmas

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ABSTRACT

The evolution of sheath in plasma contaminated with varying dust charges under the effect of an external magnetic field is studied. Study of Sagdeev potential through pseudoptential approach has been attempted with a view to deriving the sheath equation. Numerical analysis has been carried out to study the potential variation with sheath-ward distance for various plasma parameters. A unique finding of the study is that the presence of dust particles as well as the magnetic field drastically modifies the Bohm sheath criterion for plasma sheath formation as obtained earlier in unmagnetised two-component plasma. The results have more realistic interpretation in showing explicitly the interaction of magnetic field and impurity caused by dust charge variation, with the possibility of its impact in various technological applications including plasma-material interaction, material processing and electro-mechanical devices.

1. Introduction

The presence of charged dust grains in plasma along with their interaction mechanisms play a vital role in grasping the physics behind various observed phenomena in laboratory dusty plasma [1,2], astrophysical plasmas [3], fusion devices [4] and plasma processing [5]. Charging of dust grains in plasma up to a high value of $10^3 - 10^5$ electronic charges are carried out by various processes depending upon local plasma environment including plasma currents, photo electric effect, secondary emission. Theoretical prediction of the existence of very-low-velocity dust acoustic waves (DAW) in an unmagnetized plasma was made by Rao et al. In earlier studies, the charge of the massive dust grains was considered to be constant [6–11]. But, as a matter of fact, the dust charge is subject to fluctuation [12,13] owing to local plasma currents flowing onto the grains.

Dynamical properties of random charge fluctuations in a dusty plasma with different charging mechanisms have been studied by various authors [14–16]. Maitra [17] observed that dust size distribution and dust charge variation have significant effects on the characteristics of dust acoustic solitary waves in a magnetised plasma. Bhattacharjee and Das [18] have shown that charging dynamics play an effective role in modification of plasma dielectric response function. The study of dust charge fluctuation is important in the light of the fact that a number of phenomena like charge fluctuation induced heating of dust grains [19], electrostatic wave damping [20], shock wave formation [21], etc. are observed due to its effect.

With a view to analysing the importance of the plasma sheath [22,23], which is basically a non-neutral layer, with thickness of few Debye lengths in the presence of magnetic field, a number of studies [24–26] have been conducted. Tsui et al. [27] have devised a

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Child-Langmuir law-based method for accounting for Debye sheath expansion while fitting the current-voltage I-V characteristic of proud Langmuir probes (electrodes that extend into the volume of the plasma). Chodura [28] is credited with the introduction of a hydrodynamic model for semi-infinite plasma, which is valid for an oblique magnetic field to the wall, without taking into account the effect of ionization and collisions. Khoramabadi et al. [29] have conducted a study of the ion temperature effect on magnetized plasma sheath. Their study has revealed that the ion temperature strongly affects the sheath properties, i.e., an increase of the ion temperature leads to a slow increase of the sheath width and to a decrease of the potential at the sheath edge. The study was conducted for an electron-ion plasma (only one species of ion). Subsequently Hatami et al. [24] have extended the study of magnetized plasma to two species of positive ions. They have pointed out that by increasing the ion-neutral collisional frequency, the amplitude of ion density fluctuation and velocity increases.

Almost all the studies carried out earlier on magnetised plasma sheath have ignored the variation of dust charge. In this backdrop, the present work is an attempt to study the generation of sheath in plasma containing dust grains with varying charges and under the effect of an external magnetic field as may be observed in laboratory plasma.

2. Basic equations and derivation of nonlinear wave equation

In the present study, we have considered an isothermal plasma comprising electrons and ions contaminated with micron sized massive dust grains under the assumption [30] $T_\beta(\beta = e, i) >> T_d$, embedded in an inclined magnetic field. The unidirectional acoustic wave is propagating along x - direction, whereas the magnetic field H_0 is in the x-z plane.

The basic equations governing the plasma dynamics in fluid approximation, viz. the equations of continuity and motion, are written in the following normalised form [31,32].

$$\frac{\partial n_d}{\partial t} + \frac{\partial (n_d v_x)}{\partial x} = 0 \tag{1}$$

$$\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} = \frac{Z_d}{\alpha^2}\frac{\partial \phi}{\partial x} - \frac{Z_d v_y \sin\theta}{\alpha} \tag{2}$$

$$\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} = \frac{Z_d v_x \sin\theta}{\alpha} - \frac{Z_d v_z \cos\theta}{\alpha} \tag{3}$$

$$\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} = \frac{Z_d v_y \cos\theta}{\alpha} \tag{4}$$

where n_d is the number density of the dust particle moving with velocity $v_d(v_x, v_y, v_z)$ and $n_j(j = e, i)$ represents the number densities of electrons and ions, θ is the angle made by the direction of the applied magnetic field with the propagation direction. Without loss of generality, we assume that the dust grains are spherical in shape of radius 'a' and the surface charge is denoted by $q_d = -Z_d e$.

The normalisation factors are: $x = \frac{\bar{x}}{\rho}$ with $\rho = \frac{c_d}{\alpha w_d}$; $t = \frac{\bar{t}}{(\alpha\omega_d)^{-1}}$ with $\omega_d = \frac{eZ_{d0}H_0}{cm_d}$; $\alpha^2 = \frac{1}{(\gamma\delta_1+\delta_2)(\delta_1-\delta_2)}$ with $\delta_1 = \frac{n_{i0}}{n_{d0}Z_{d0}}$, $\delta_2 = \frac{n_e0}{n_{d0}Z_{d0}}$; $\gamma = \frac{T_e}{T_i}$; $n_j = \frac{\overline{n_j}}{n_{j0}}(j = d, i, e)$; $\phi = \frac{e\bar{\phi}}{KT_e}$ where m_d, m_e, m_i are the masses of dust particles, ions and electrons.

Since our interest is to study the DAW, we have considered that on the inertial time scale of cold but heavier dust charged grains, electrons and ions are represented by Bolzmannian relations defined through their densities as [33]:

$$n_i = \exp(-\gamma\phi) \qquad \& \qquad n_e = \exp(\phi) \tag{5}$$

As a result, the Poisson's equation assumes the form

$$\frac{\lambda_d^2}{\rho^2}\frac{\partial^2\phi}{\partial x^2} = n_d Z_d + \delta_2 \exp\phi - \delta_1 \exp(-\gamma\phi) \tag{6}$$

where $\lambda_d = \sqrt{\frac{kT_e}{4\pi e^2 n_{d0}Z_{d0}}}$ is the Debye length. The basic equations are supplemented by the charge neutrality condition expressed through the relation $\delta_1 - \delta_2 = 1$. We have considered an uniform dust charge fluctuation and charging equations due to plasma currents are taken as

$$\frac{dZ_d}{dt} = \frac{\alpha\omega_d n_{d0}}{eZ_{d0}}[I_i + I_e] \tag{7}$$

with

$$I_i = \pi a^2 e\sqrt{\frac{8T_i}{\pi m_i}}\,\delta_1 \exp(-\gamma\phi)\left(1 - \frac{e^2 Z_d Z_{d0}}{aT_i}\right)$$

and

$$I_e = \pi a^2 e\sqrt{\frac{8T_e}{\pi m_e}}\,\delta_2 \exp(\phi)\exp\left(\frac{e^2 Z_d Z_{d0}}{aT_e}\right)$$

under the appropriate condition $a << \lambda_d << \lambda_{mfp}$ (where λ_d is the plasma Debye length, and λ_{mfp} is the is the mean free path for ion and electron collision). The orbital - motion - limited theory [34] describes the current generation on dust grains. It is also found that for ρ > > a(where ρ is the mean gyroradius), the effect of magnetic field on charging dust is unimportant [34,35]. Under this broad assumption and based on earlier works, the above charging mechanism has been imposed in the present model of dusty plasma. In order to solve Eqs. (1)–(7), we first take the frame of reference by transforming the coordinates through a Galilean transformation $\xi = x - Mt$, with respect to a frame moving with a velocity M. As a result, Eqs. (1)–(4) are reduced to the following forms:

$$-M\frac{dn_d}{d\xi} + \frac{d}{d\xi}(n_d v_x) = 0 \tag{8}$$

$$-M\frac{dv_x}{d\xi} + v_x\frac{dv_x}{d\xi} = \frac{Z_d}{\alpha^2}\frac{d\phi}{d\xi} - \frac{Z_d v_y \sin\theta}{\alpha} \tag{9}$$

$$-M\frac{dv_y}{d\xi} + v_x\frac{dv_y}{d\xi} = \frac{Z_d v_x \sin\theta}{\alpha} - \frac{Z_d v_z \cos\theta}{\alpha} \tag{10}$$

$$-M\frac{dv_z}{d\xi} + v_x\frac{dv_z}{d\xi} = \frac{Z_d v_y \cos\theta}{\alpha} \tag{11}$$

Integrating Eqs. (8),(9) &–(10) and using the appropriate boundary conditions $v_x \to 0$, $\phi \to 0$, $\frac{d\phi}{d\xi} \to 0$ and $n_d \to 1$ at $\xi \to \pm\infty$, we get

$$v_x = M\left(1 - \frac{1}{n_d}\right) \tag{12}$$

$$v_y = \frac{1}{\alpha\sin\theta}\left[1 + \frac{\alpha^2 M^2}{Z_d n_d^3}\frac{dn_d}{d\phi}\right]\frac{d\phi}{d\xi} \tag{13}$$

$$v_z = M cot\theta\left(\frac{1}{n_d} - 1\right) - \frac{cot\theta}{\alpha^2 M}\int_0^{\phi} n_d Z_d d\phi \tag{14}$$

From which after using Eqs. (12), (13) &–(14) in Eq. (10), we derive the desired modified Sagdeev Potential equation as

$$\frac{d}{d\xi}\left[A(n_d, Z_d)\frac{d\phi}{d\xi}\right] = Z_d - n_d Z_d - \frac{n_d Z_d Cos^2\theta}{\alpha^2 M^2}\int n_d Z_d d\phi = -\frac{dV}{d\phi} \tag{15}$$

with $A(n_d, Z_d) = 1 + \frac{\alpha^2 M^2}{n_d^3 Z_d}\frac{dn_d}{d\phi}$ where V is defined as the classical potential. Here $A(n_d, Z_d)$ plays the main role in exhibiting the solution of Eq. (15) and thus the potential variation in dusty plasma. Now Eq. (15) is equivalent to the sheath equation considered by Chen [36]. For further analysis, small amplitude wave approximation as well as the quasineutrality condition in plasma is taken into consideration on the basis of the assumption that electron and ion Debye lengths are much smaller in comparison to the dust gyroradius, from which we have:

$$n_d Z_d = \delta_1 n_i - \delta_2 n_e = \delta_1\exp(-\gamma\phi) - \delta_2\exp(\phi) \tag{16}$$

Again from Eq. (7) we derive

$$\frac{dZ_d}{d\phi}\frac{d\phi}{d\xi} = \frac{\pi a^2\alpha\omega_d n_d n_{d0}}{MZ_{d0}}\sqrt{\frac{8}{\pi}}\sqrt{\frac{T_e}{m_e}}\exp\left(\phi + \frac{e^2 Z_d Z_{d0}}{aT_e}\right) - \frac{\pi a^2\alpha w_d n_d n_{d0}}{MZ_{d0}}\sqrt{\frac{8}{\pi}}\sqrt{\frac{T_i}{m_i}}\exp(-\gamma\phi)\left(1 - \frac{e^2 Z_d}{\alpha T_i}\right) \tag{17}$$

From Eq. (17) it is worth distinguishing between the time scales for grain charging and dust hydrodynamic motion. It is observed that the dust charging time $\tau_{ch} < < \tau_d$, i.e. dynamical time at which $I_i + I_e \sim 0$ [37] and using which Z_d has been obtained as

$$Z_d = 1 - (1 - \gamma^2)\phi \tag{18}$$

with $Z_{d0} = \frac{\alpha T_e}{e^2}A(n_d, Z_d)$ can be simplified as

$$A(n_d, Z_d) = 1 - \frac{\alpha^2 M^2}{(n_d Z_d)^3}\left[Z_d\frac{d}{d\phi}n_d Z_d - n_d Z_d\frac{d}{d\phi}Z_d\right] \tag{19}$$

Finally Eq. (15) can be expressed as

$$\frac{dW}{d\xi} = \frac{Z_d - n_d Z_d - \frac{n_d Z_d\cos^2\theta}{\alpha^2 M^2}\int n_d Z_d d\phi}{A} - \frac{\frac{dA}{d\phi}W^2}{A} \tag{20}$$

with

$$\frac{d\phi}{d\xi} = W \tag{21}$$

and thus

$$\frac{dW}{d\xi} = \frac{d^2\phi}{d\xi^2} \tag{22}$$

For lowest order approximation of $A(n_d, Z_d)$, Eq. (20) finally reduces to

$$\frac{1}{2}A\left[\frac{d\phi}{d\xi}\right]^2 = \phi - \frac{(1-\gamma^2)\phi^2}{2} + \frac{\delta_1 e^{-\gamma\phi}}{\gamma} - \delta_2 e^{\phi} - \frac{\cos^2\theta}{\alpha^2 M^2}\left[\frac{\delta_1^2}{-2\gamma}e^{-2\gamma\phi} + \delta_1\delta_2\left(\frac{1}{\gamma}-1\right)(1-\gamma)e^{(1-\gamma)\phi} + \frac{\delta_2^2 e^{2\phi}}{2}\right] \tag{23}$$

with

$$A(n_d, Z_d) = 1 - \gamma^2 M^2(1-\gamma^2) + \alpha^2 M^2(\gamma\delta_1 + \delta_2) \tag{24}$$

Eqs. (23) and(24), with the use of suitable boundary conditions, form the foundation for discussing the formation and the characteristic behaviour of sheath in plasma. At the edge of the sheath, ϕ should be minimum so that a smooth, non-oscillatory and monotonic transition of plasma from pre-sheath to sheath region takes place. The condition for minimising the potential at the sheath edge, $\phi = \phi_m$,requires the following derived condition

$$\frac{d^2\phi}{d\xi^2} \geq 0 \tag{25}$$

Thus from Eq. (20), we get

$$Z_d - n_d Z_d - n_d Z_d\frac{\cos^2\theta}{\alpha^2 M^2}\int n_d Z_d d\phi - \frac{dA}{d\phi}\left(\frac{d\phi}{d\xi}\right)^2 \geq 0 \tag{26}$$

The boundary conditions $\phi \to 0$, $n_d \to 1$ and $\frac{d\phi}{d\xi} \neq 0$ hold good at the sheath edge. Thus applying the condition for monotonic potential transition in Eq. (26), the Bohm condition for sheath formation [38,39] comes out as $M > M_c$ where

$$M_c = \sqrt{\frac{(\gamma\delta_2 + \delta_1)}{3\gamma\{1-(1-\gamma^2)\alpha^2\}}\frac{\cos\theta}{d\phi/d\xi}} \tag{27}$$

Condition (27) is the modified Bohm criterion in magnetised dusty plasma with dust charge fluctuation under consideration. It is clearly evident that the original Bohm condition for the formation of plasma sheath which requires that the minimum threshold value of the ion Mach number should be unity gets significantly modified in the presence of dust particles as well as the magnetic field. In the present study, the critical Mach number M_c undergoes a change due to the joint role of magnetic field via θ, temperature ratio via $\gamma\left(=\frac{T_e}{T_i}\right)$ and number densities of electrons and ions represented by $\delta_2\left(=\frac{n_{e0}}{n_{d0}z_{d0}}\right)$ and $\delta_1\left(=\frac{n_{i0}}{n_{d0}z_{d0}}\right)$ respectively. It is worth mentioning here that Mishra [40] studied the collisional plasma and showed that the thickness of the sheath is substantially greater than the Debye length and at the edge of the sheath, the critical drift velocity of plasma is smaller than the Bohm velocity.

3. Results and discussion

Integration of Eqs. (20) and (21) have been carried out numerically to determine the potential of the sheath. At the initial point $\xi = \xi_0$, $\phi = 0$ has been assumed and $\frac{d\phi}{d\xi}$ takes on a finite value. At the wall, i.e. at ξ_w, where $\xi = 0$, the total current dies out. Under the consideration of the velocity of a Boltzmannian species at the wall being the first moment of half Maxwellian distribution and the conservation of dust flux, the condition that can be applied at the wall is

$$n_{i0}\exp\left(-\frac{e\phi_w}{T_i}\right)\left(\frac{T_i}{2\pi m_i}\right)^{\frac{1}{2}} = Z_d n_{d0} v_{d0} + n_{e0}\exp\left(\frac{e\phi_w}{T_e}\right)\left(\frac{T_e}{2\pi m_e}\right)^{\frac{1}{2}} \tag{28}$$

with v_{d0} as the dust fluid velocity of the main plasma. Since $v_{d0} \sim c_d$ for Mach number M $\sim$ 1, we obtain

$$\phi_w = -\frac{1}{1+\gamma}\left[2\ln Z_d - \frac{3}{2}\ln\delta_2 + \ln\delta_1 + \frac{1}{2}\ln\frac{T_i}{m_d} + \frac{1}{2}\ln\left(\frac{2\pi m_e}{T_e}\right) - \frac{1}{2}\ln\left(\gamma\frac{m_e}{m_i}\right)\right] \tag{29}$$

with dust acoustic speed $c_d = Z_d\sqrt{(\frac{n_{d0}}{n_{i0}})(\frac{T_i}{m_d})}$ for $\lambda_{De}^2 >> \lambda_{Di}^2$. Eq. (29) shows that the wall potential can take on both positive and negative values with respect to the main body of the plasma. The reason is the mobility of the positive ions and the electrons due to which they can rush off to the wall together. The relative consideration of the ions and electrons is the determining factor of the sign of the wall potential. Integrating Eq. (22) in which lowest order approximation of $A(n_d, Z_d)$ has been considered, the starting point is $\xi_w = 0$ i.e the wall where ϕ and $\frac{d^2\phi}{d\xi^2}$ are finite. ξ_0 can be calculated using the condition that $\phi \to 0$, $\frac{d\phi}{d\xi} \to 0$, $\frac{d^2\phi}{d\xi^2} \to 0$ at $\xi = \infty$.

In Fig. 1, we plot the variation of the normalised electrostatic potential ϕ with normalised distance ξ for typical values of plasma parameters M = 1.5, $\theta = 60°$, $\delta_1 = 1.01$ and $\delta_2 = 0.01$ with different values of temperature ratio γ.

For Curve 1, $\gamma = 1.5$; for Curve 2, $\gamma = 3.5$ and for Curve 3, $\gamma = 5.0$. The relative concentration of the positive ions and electrons decide whether the wall potential is positive or negative. Here we have considered the more realistic case of negative wall potential. It is evident from the plot that the sheath width shows a decreasing trend as temperature ratio increases. However, an interesting feature that comes to light is that the rate of decrease of the sheath width slows down as the value of γ increases. The ions shield the

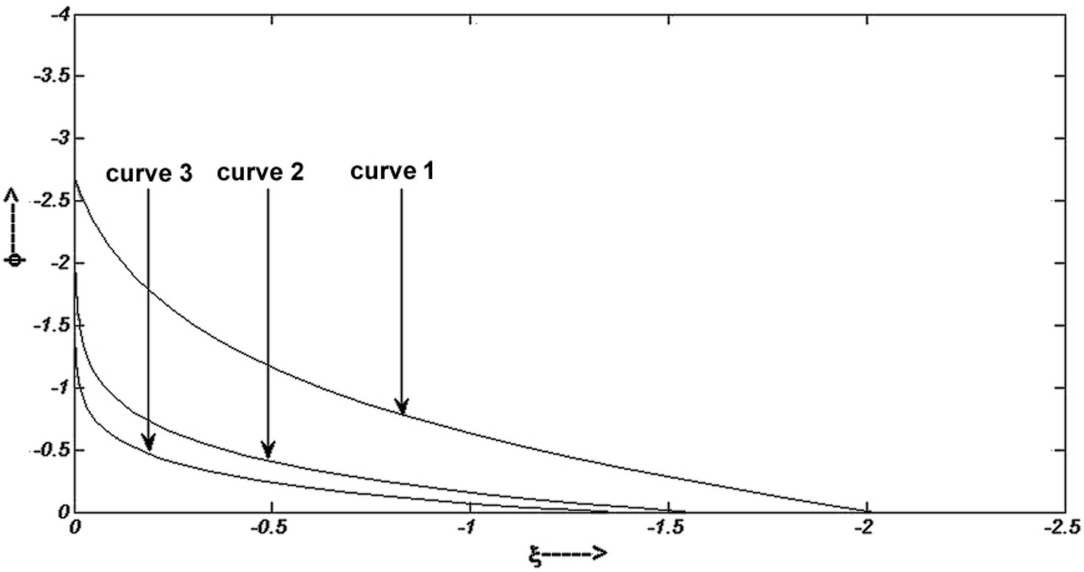

Fig. 1. Variation of the normalised electrostatic potential (ϕ) with normalised distance (ξ) for $\gamma = 1.5$ (Curve 1), $\gamma = 3.5$ (Curve 2) and $\gamma = 5.0$ (Curve 3). The parameters are M = 1.5, $\theta = 60°$, $\delta_1 = 1.01$ and $\delta_2 = 0.01$.

wall from the bulk plasma in this case of negative wall potential.

We then proceed to study the variation of the normalised electrostatic potential ϕ with normalised distance ξ for different values of δ with plasma parameters M = 1.5, $\theta = 60°$, $\gamma = 3.5$, and the results are depicted in Fig. 2.

In case of Curve 1, $\delta_1 = 1.01$ and $\delta_2 = 0.01$; for Curve 2, $\delta_1 = 1.1$ and $\delta_2 = 0.1$, whereas for Curve 3, $\delta_1 = 1.5$ and $\delta_2 = 0.5$. It is found that the sheath thickness is greater for a lower value of δ_1 and vice versa. It shows that ion and electron concentrations have a significant effect on the thickness of the sheath. The trend of the curves show that the sheath potential is negative and the width of the sheath increases with smaller ion and electron concentrations.

Fig. 3 shows the variation of the normalised electrostatic potential ϕ with normalised distance ξ for different values of Mach Number M. The typical plasma parameters considered for the plot are $\theta = 80°$, $\gamma = 3.5$, $\delta_1 = 1.0001$ and $\delta_2 = 0.0001$.

For Curve 1, M is taken as 1.1, for Curve 2, M = 1.3 and for Curve 3, M is 1.5. The plot explicitly shows that the sheath thickness increases with rise in the value of M. When the velocity of the plasma increases as a result of increase of Mach number, the dust charged grains drag along with them the positively charged ions; the shielding effect thereby reduces and so, the thickness of the

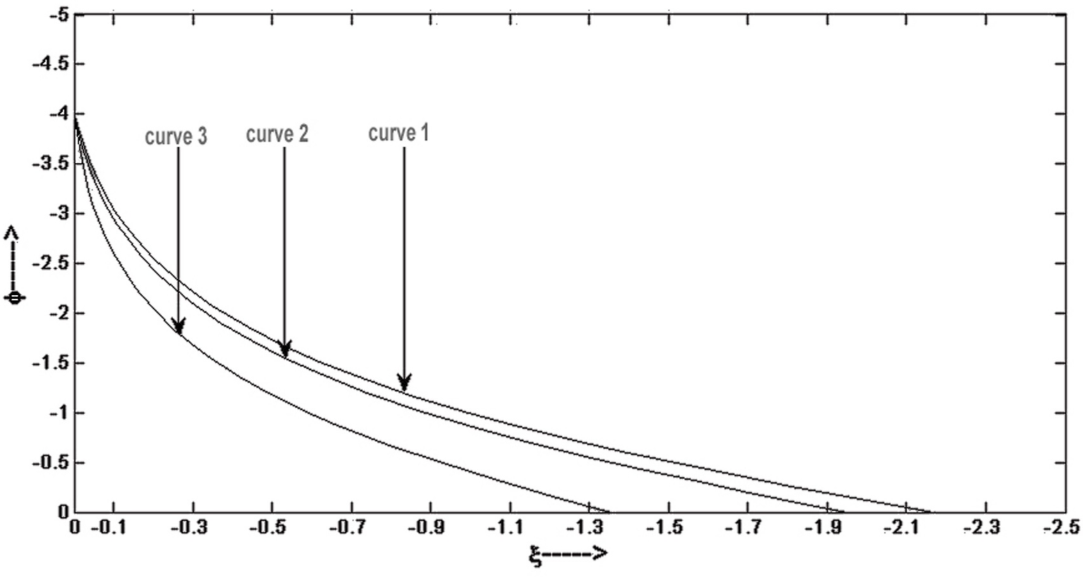

Fig. 2. Variation of the normalised electrostatic potential (ϕ) with normalised distance (ξ) for $\delta_1 = 1.01$ and $\delta_2 = 0.01$ (Curve 1);, $\delta_1 = 1.1$ and $\delta_2 = 0.1$ (Curve 2); $\delta_1 = 1.5$ and $\delta_2 = 0.5$ (Curve 3). The parameters are M = 1.5, $\theta = 60°$, $\gamma = 3.5$.

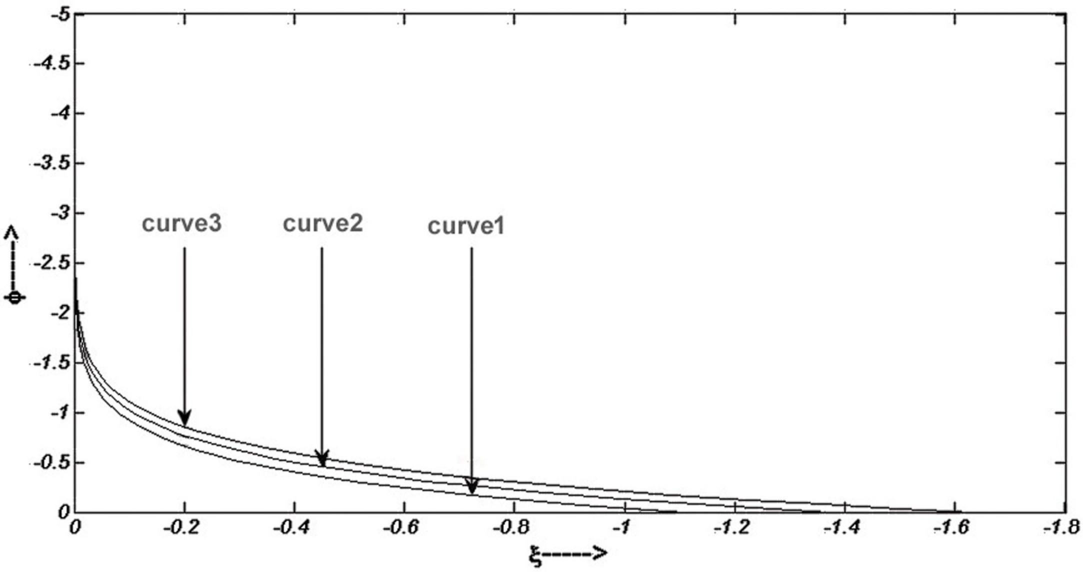

Fig. 3. Variation of the normalised electrostatic potential (ϕ) with normalised distance (ξ) for M = 1.1 (Curve 1), M = 1.3(Curve 2)and M = 1.5 (Curve 3). The parameters are $\theta = 80°$, $\gamma = 3.5$, $\delta_1 = 1.0001$ and $\delta_2 = 0.0001$.

sheath increases.

In Fig. 4, the variation of the normalised electrostatic potential ϕ with normalised distance ξ for a changing magnetic field strength, which has been effected by the variation of the inclination θ of the magnetic field, is depicted.

In this plot, the typical plasma parameters considered are $M = 1.5$, $\gamma = 1.1$, $\delta_1 = 1.001$ and $\delta_2 = 0.001$. For $\theta = 10°$ (Curve 1), $\theta = 30°$ (Curve 2) and $\theta = 60°$ (Curve 3), a increasing trend of sheath thickness is visible with increase of θ. As the inclination of the magnetic field with the propagation direction is raised, the dust charged grains, in addition to the charge fluctuations, undergo a greater $E \times B$ drift. The consequence is that the dust charge grains carry along with them the oppositely charged ions thereby weakening the shielding and as such, the sheath thickness becomes wider.

4. Conclusion

To conclude with, it may be said that the present study has encompassed the generation of sheath in a plasma containing charge varying dust grains and under the effect of an inclined magnetic field. Investigations have revealed that the presence of dust grains with varying charges can, to a considerable extent, affect the generation as well as the characteristic behaviour of the plasma sheath. The studies bring to light the fact that the electron-ion temperature ratio, electron as well as ion concentrations, Mach number and the inclination of the magnetic field with the direction of propagation of plasma have marked effects on the thickness of the sheath. Another notable outcome of the study is that the Bohm sheath criterion in earlier studies carried out in unmagnetised plasma undergoes a notable change under the combined effect of the presence of dust particles as well as the magnetic field. It is also found that with a greater inclination of the magnetic field to the propagation direction, the dust charged grains suffer a greater $E \times B$ drift.

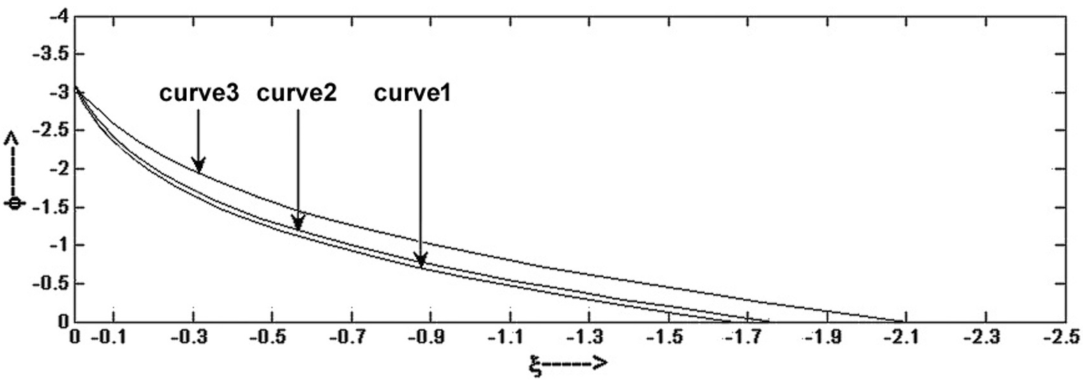

Fig. 4. Variation of the normalised electrostatic potential (ϕ) with normalised distance (ξ) for $\theta = 10°$ (Curve 1), $\theta = 30°$ (Curve 2) and $\theta = 60°$ (Curve 3). The parameters are M = 1.5, $\gamma = 1.1$, $\delta_1 = 1.001$ and $\delta_2 = 0.001$.

The new observations emanating from these studies might be helpful in view of the possibility of its impact in various technological applications, such as plasma-material interaction, material processing, electro-mechanical devices and so forth [41,42].

Declaration of Competing Interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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